

II. *Æquationum Cubicarum & Biquadraticarum, tum Analytica, tum Geometrica & Mechanica, Resolutio Universalis, a J. Colson.*

§. I. *Æquationis Cubicæ* $\left\{ \begin{array}{l} x^3 = 3 p x^2 + 3 q x + 2 r, \\ \text{Universalis} \quad \quad \quad - 3 p^2 + p^3 \\ \quad \quad \quad \quad \quad \quad - 3 p q \end{array} \right.$

Radices Tres sunt,

$$x = p + \sqrt[3]{r + \sqrt{r^2 - q^3}} + \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

$$x = p - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

$$x = p - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

Vel ut Calculus Arithmeticus facilius ac paratior evadat, si posueris Binomii irrationalis $r + \sqrt{r^2 - q^3}$ Radicem Cubicam esse $m + \sqrt{n}$, erunt ejusdem *Æquationis* Radices tres $x = p + 2 m$, & $x = p - m \pm \sqrt{-3 n}$.

Igitur data *Æquatione* quavis Cubica, inter ejus hujusque *Æquationis* Universalis terminos singulos instituenda est comparatio, quo pacto facillime invenientur ipsæ p , q , r ; & hisce cognitis, innotescunt *Æquationis* datæ Radices omnes. Hujus vero Solutionis Exempla sint sequentia in Numeris.

I. *Æquationis* Cubicæ $x^3 = 2 x^2 + 3 x + 4$ sit Radix x indaganda. Erit primò juxta præscriptum $3 p = 2$,
14. N five

five $p = \frac{2}{3}$. Secundò $3q - (3p^2) \frac{4}{3} = 3$, five $q = \frac{13}{9}$.

Tertiò $2r (+ \overline{p^2 - 3q \times p}) - \frac{70}{27} = 4$, five $r = \frac{89}{27}$,

& $r^2 - q^3 = \frac{212}{27}$. Et propterea $x = \frac{2}{3} + \sqrt[3]{\frac{89}{27} + \sqrt{\frac{212}{27}}}$

+ $\sqrt[3]{\frac{89}{27} - \sqrt{\frac{212}{27}}}$. Reliquæ duæ Radices sunt impossibiles.

2. In Equatione $x^3 = 12x^2 - 41x + 42$, erit primò $3p = 12$, five $p = 4$. Secundò $3q - (3p^2) 48 = -41$, five $q = \frac{7}{3}$. Tertiò $2r + (\overline{p^2 - 3q \times p}) 36 = 42$,

five $r = 3$; Et inde $r^2 - q^3 = -\frac{100}{27}$. At Binomii furdi

$3 + \sqrt{-\frac{100}{27}} (= r + \sqrt{r^2 - q^3})$ Radix Cubica, per Methodos ex Arithmetica petendas extracta, est $-1 +$

$\sqrt{-\frac{4}{3}} (= m + \sqrt{n},)$ & proinde Radix $x = (p + 2m = 4 - 2 =) 2$, vel etiam $x = (p - m + \sqrt{-3n} = 4 + 1 + (\sqrt{4})^2 =) 7$ vel 3. Vel rursus, ejusdem Binomii

$3 + \sqrt{-\frac{100}{27}}$. Radix alia Cubica (tres enim agnoscit)

est $\frac{3}{2} + \sqrt{-\frac{1}{12}} (= m + \sqrt{n},)$ & proinde Radix

$x = (p + 2m = 4 + 3 =) 7$, & etiam $x = (p - m + \sqrt{-3n} = 4 - \frac{3}{2} + (\sqrt{\frac{1}{4}}) \frac{1}{2} =) 3$ vel 2. Vel denuo,

ejusdem Binomii $3 + \sqrt{-\frac{100}{27}}$ Radix Cubica tertia est

$-\frac{1}{2} + \sqrt{-\frac{25}{12}}, (= m + \sqrt{n},)$ & proinde Radix

$x =$

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$x = (p + 2m = 4 - 1 =) 3$, atque etiam $x = (p - m \pm \sqrt{1 - 3n = 4 + \frac{1}{2} \pm (\sqrt{\frac{25}{4}}) \frac{5}{2}} =) 7$ vel 2.

3. In *Æquatione* $x^3 = -15x^2 - 84x + 100$, erit $p = -5$, $q = -3$, $r = 135$; & Binomii $135 + \sqrt{18252}$ Radix Cubica est $3 + \sqrt{12}$. Igitur Radix $x = -5 + 6 = 1$, & $x = -5 - 3 \pm \sqrt{-36} = -8 \pm \sqrt{-36}$, impossibiles.

4. In *Æquatione* $x^3 = 34x^2 - 310x + 1012$, erit $p = \frac{34}{3}$, $q = \frac{226}{9}$, $r = \frac{5536}{27}$; & Binomii $\frac{5536}{27} + \sqrt{\frac{707560}{27}}$ Radix Cubica est $\frac{16}{3} + \sqrt{\frac{10}{3}}$. Igitur Radix $x = \frac{34}{3} + \frac{32}{3} = 22$, & $x = \frac{34}{3} - \frac{16}{3} \pm \sqrt{-10} = 6 \pm \sqrt{-10}$, impossibiles.

5. In *Æquatione* $x^3 = 28x^2 + 61x - 4048$, erit $p = \frac{28}{3}$, $q = \frac{967}{9}$, $r = -\frac{25010}{27}$; & Binomii $-\frac{25010}{27} + \sqrt{-382347}$. Radix Cubica est $\frac{41}{6} + \sqrt{-\frac{243}{4}}$. Igitur $x = \frac{28}{3} + \frac{41}{3} = 23$, & $x = \frac{28}{3} - \frac{41}{6} \pm (\sqrt{\frac{729}{4}}) \frac{27}{2} = 16$ vel -11 .

6. In *Æquatione* $x^3 = -x^2 + 166x - 660$, erit $p = -\frac{1}{3}$, $q = \frac{499}{9}$, $r = -\frac{9658}{27}$; & Binomii $-\frac{9658}{27} + \sqrt{-\frac{1147205}{27}}$ Radix Cubica est $-\frac{22}{3} + \sqrt{-\frac{5}{3}}$. Igitur $x = -\frac{1}{3} - \frac{44}{3} = -15$, & $x = -\frac{1}{3} + \frac{22}{3} \pm \sqrt{5} = 7 \pm \sqrt{5}$, irrationales.

7. In

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7. In *Æquatione* $x^3 = 63 x^2 + 99673 x + 9951705$,
erit $p = 21$, $q = \frac{100996}{3}$, $r = 6031680$; & Binomii
 $6031680 + \sqrt{\frac{47887175043136}{27}}$ Radix Cubica est

$183 + \sqrt{\frac{529}{3}}$. Igitur $x = 21 + 366 = 387$, &
 $x = 21 - 183 \pm (\sqrt{529}) 23 = -139$ vel 185 .

Nec secus in cæteris procedendum: Investigatur autem
Theorema ad modum sequentem. Pono *Æquationis* cu-
jusdam Cubicæ Radicem $z = a + b$, & cubicè multi-
plicando proveniet $z^3 = (a^3 + 3 a^2 b + 3 a b^2 + b^3 =)$
 $a^3 + 3 a b \times a + b + b^3$. Jam loco ipsius $a + b$ valo-
rem ejus z substituendo, fiet $z^3 = 3 a b z + a^3 + b^3$, quæ
est *Æquatio Cubica ex Radice* $z = a + b$ constructa, cui
terminus secundus deest. Ut hæc verò ad formam magis
commodam magisq; concinnam revocenter, sumo *Æqua-*
tionem $z^3 = 3 q z + 2 r$, quæ posthac ipsius $z^3 = 3 a b z$
 $+ a^3 + b^3$ vices gerat. Igitur transmutatione hujus in
illam, fiet primò $3 q = 3 a b$, five $q_3 = a^3 b^3$; & se-
cundò $2 r = a^3 + b^3$, five $2 r a^3 = (a^6 + a^3 b^3 =) a^6 + q^3$.
Et soluta hac *æquatione quadratica*, erit $a^3 = r + \sqrt{r^2 - q^3}$,
& hinc $b^3 = (2 r - a^3 =) r - \sqrt{r^2 - q^3}$: Atque igi-

tur tandem $a = \sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $b = \sqrt[3]{r - \sqrt{r^2 - q^3}}$.
Et propterea in *Æquatione Cubica* $z^3 = 3 q z + 2 r$ erit

Radix $z = (a + b =) \sqrt[3]{r + \sqrt{r^2 - q^3}} + \sqrt[3]{r - \sqrt{r^2 - q^3}}$

At verò hæc Radix reverà triplex est, pro triplici va-
lore quem induere potest & $\sqrt[3]{r + \sqrt{r^2 - q^3}}$ &

$\sqrt[3]{r - \sqrt{r^2 - q^3}}$. Cujusvis enim quantitatis Radix Cu-
bica triplex erit, & ipsius Unitatis Radix Cubica vel
est

est 1, vel $-\frac{1}{2} + \frac{1}{2}\sqrt{-3}$, vel $-\frac{1}{2} - \frac{1}{2}\sqrt{-3}$:

Atque id adeo, propterea quòd harum alicujus Cubus fit

Unitas. Igitur si $1 \times \sqrt[3]{r + \sqrt{r^2 - q^3}}$ aut $\sqrt[3]{r + \sqrt{r^2 - q^3}}$
 $(= \sqrt[3]{1 \times r + \sqrt{r^2 - q^3}} = \sqrt[3]{1} \times \sqrt[3]{r + \sqrt{r^2 - q^3}})$ Radicem aliquam [quam supra nominavimus $m + \sqrt{n}$, aut
 $1 \times m + \sqrt{n}$,] Cubi $r + \sqrt{r^2 - q^3}$ designet; ipsæ
 $\frac{-1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $\frac{-1 - \sqrt{-3}}{2}$
 $\times \sqrt[3]{r + \sqrt{r^2 - q^3}}$ [i. e. $\frac{-1 + \sqrt{-3}}{2} \times m + \sqrt{n}$ &
 $\frac{-1 - \sqrt{-3}}{2} \times m + \sqrt{n}$] alias duas ejusdem Cubi Ra-
dices designabunt. Similiter & $\sqrt[3]{r - \sqrt{r^2 - q^3}}$,
 $\frac{-1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$, & $\frac{-1 - \sqrt{-3}}{2}$
 $\times \sqrt[3]{r - \sqrt{r^2 - q^3}}$, [i. e. $m - \sqrt{n}$, $\frac{-1 + \sqrt{-3}}{2}$
 $\times m - \sqrt{n}$, $\frac{-1 - \sqrt{-3}}{2} \times m - \sqrt{n}$] tres Cubicæ Ra-
dices erunt Apotomes $r - \sqrt{r^2 - q^3}$. Atque has Radices
debitè connecendo, fiet $z = \sqrt[3]{r + \sqrt{r^2 - q^3}}$
 $+ \sqrt[3]{r - \sqrt{r^2 - q^3}}$, [i. e. $z = m + \sqrt{n} + m - \sqrt{n} = 2m$,]
 $z = \frac{-1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} + \frac{-1 - \sqrt{-3}}{2}$
 $\times \sqrt[3]{r - \sqrt{r^2 - q^3}}$, [i. e. $z = \frac{-1 + \sqrt{-3}}{2} \times m + \sqrt{n}$
 $+ \frac{-1 - \sqrt{-3}}{2} \times m - \sqrt{n} = -m + \sqrt{-3}n$,] & $z =$
 $\frac{-1 - \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} + \frac{-1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$

$$[\text{i. e. } z = \frac{-1 - \sqrt{-3}}{2} \times m + \sqrt{n} + \frac{-1 + \sqrt{-3}}{2}$$

$\times m - \sqrt{n} = -m - \sqrt{-3} n,]$ quæ tres erunt Radices $\text{\AA}quationis$ Cubicæ $z^3 = 3 qz + 2 r$. Debitæ autem connectuntur Radices istæ ad modum præcedentem, nempe quæ sic connexæ, & more vulgari in se invicem continueductæ, $\text{\AA}quationem$ $z^3 = 3 qz + 2 r$ restituant. Denique fac $z = x - p$, & $\text{\AA}quatio$ fiet $x^3 - 3 p x^2 + 3 p^2 x - p^3 = 3 q x - 3 p q + 2 r$, quæ universalis est, & cujus Radices evadunt ut supra fuerunt exhibitæ.

Hic obiter notatu dignum est, quod $\text{\AA}quationis$ Cubicæ cujuscunque Radices omnes sint possibiles & reales, quoties Binomii membrum irrationale $\sqrt{r^2 - q^3}$ impossibilitatem in se complectitur; hoc est, quoties q est quantitas affirmativa, & simul cubus ejus major est quadrato ex latere r . At si membrum istud $\sqrt{r^2 - q^3}$ sit possibile, hoc est si q sit quantitas negativa, aut etiam si affirmativæ cubus sit minor quadrato ex latere r , tunc unicam tantum agnoscit $\text{\AA}quatio$ Radicem possibilem & realem, reliquæque duæ erunt impossibiles.

In hoc Theoremate si fiat $p = 0$, hoc est, si desit $\text{\AA}quationis$ terminus secundus, tunc deventum erit ad casum Regularum quæ dicuntur *Cardani*, cujus solutio continetur in præcedentibus.

§. 2. $\text{\AA}quationis$ Biquadraticæ Universalis

$$x^4 = 4 p x^3 + 2 q x^2 + 8 r x + 4s, \\ - 4 p^2 - 4 p q - q^2$$

$$\text{Radices quatuor sunt } x = p - a \pm \sqrt{p^2 + q - a^2} - \frac{2r}{a},$$

$$\& x = p + a \pm \sqrt{p^2 + q - a^2} + \frac{2r}{a}, \text{ Ubi } a^2 \text{ est Radix}$$

$$\text{\AA}quationis Cubicæ } a^6 = p^2 a^4 - 2 p r a^2 + r^2 \\ + q - s$$

Jam data $\text{\AA}quatione$ quavis Biquadratica, inter ejus hujusque $\text{\AA}quationis$ Universalis terminos singulos institu-
enda

enda est comparatio, quo pacto citissime invenientur ipsæ p, q, r, s; & hisce cognitis, non latebit valor ipsius a, ex Theoremate superiori inveniendus, & tum demum innotescant Æquationis datæ Radices omnes.

Huic Solutioni illustrandæ Exemplum unum aut alterum sufficiat.

1. Æquationis Biquadraticæ $x^4 = 8x^3 + 83x^2 - 162x - 936$ sint Radices extrahendæ. Erit primò juxta præscriptum $4p = 8$, five $p = 2$. Secundò $2q - (4p^2)$ $16 = 83$, five $q = \frac{99}{2}$. Tertiò $8r - (4pq)$ $396 = -162$, five $r = \frac{117}{4}$. Quartò $4s - (q^2)$ $\frac{9801}{4} = -936$, five $s = \frac{6057}{16}$. Hinc $p^2 + q = \frac{107}{2}$, $2pr + s = \frac{7929}{16}$, $r^2 = \frac{1368}{19}9$, & proinde $a^6 = \frac{107}{2}a^4 - \frac{7929}{16}a^2 + \frac{1368}{16}9$. Jam ut Æquatio hæc aliquatenus Cubica in Radices ejus resolvatur, ad Theorema præcedens recurrendum est, in quo erit $p = \frac{107}{2}$, $q = \frac{22009}{144}$, $r = \frac{2903923}{1728}$ & $r^2 - q^3 = -\frac{11940075}{16}$. Atqui Binomii $\frac{2903923}{1728} + \sqrt{-\frac{11940075}{16}}$ Radix Cubica est $-\frac{53}{12} + \sqrt{-\frac{400}{3}}$ & propterea $a^2 = \frac{107}{6} - \frac{53}{6} = 9$, & etiam $a^2 = \frac{107}{6} + \frac{53}{12} \pm (\sqrt{400}) 20 = \frac{169}{4}$ vel $\frac{2}{4}$; Vel quod perinde est, Æquationis præmissæ reverà Cubo-Cubicæ sex Radices sunt $a = \pm 3$, $a = \pm \frac{13}{2}$, & $a = \pm \frac{3}{2}$, quarum quævis indiscriminatim proposito

fito nostro faciet satis. Puta si in præsentī casu fiat $a = 3$, erit juxta Theorema $x = \frac{(p - a + \sqrt{p^2 + q - a^2 - \frac{2r}{a}})}{2} = \frac{2 - 3 \pm \sqrt{4 + \frac{99}{2} - 9 - \frac{39}{2}}}{2} = \frac{-1 + (\sqrt{25}) 5}{2} = 4$ vel -6 , & $x = \frac{(p + a + \sqrt{p^2 + q - a^2 + \frac{2r}{a}})}{2} = \frac{2 + 3 \pm \sqrt{4 + \frac{99}{2} - 9 + \frac{39}{2}}}{2} = \frac{5 \pm (\sqrt{64}) 8}{2} = 13$ vel -3 , quæ sunt Æquationis datæ Radices quatuor,

2. In Æquatione $x^4 = 20x^3 + 252x^2 - 6592x + 21312$, erit $p = 5$, $q = 176$, $r = -384$, & $s = 13072$. Hinc $p^2 + q = 201$, $2pr + s = 9232$, & $r^2 = 147456$; & inde $a^6 = 201 a^4 - 9232 a^2 + 147456$.

Jam in Theoremate pro Cubicis erit $p = 67$, $q = \frac{4235}{3}$, & $r = 65219$; eritque Binomii $65219 + \sqrt{\frac{38889307072}{27}}$

Radix Cubica $\frac{77}{2} + \sqrt[3]{\frac{847}{12}}$. Igitur $a^2 = 67 + \frac{77}{2} = 144$, five $a = 12$; & proinde $x = \frac{5 - 12 \pm \sqrt{25 + 176 - 144 + 64}}{2} = \frac{-7 \pm (\sqrt{121}) 11}{2} = 4$ vel -18 , & $x = \frac{5 + 12 \pm \sqrt{25 + 176 - 144 - 64}}{2} = \frac{17 \pm \sqrt{-7}}{2}$, impossibiles.

Hujus autem Theorema is Inventio est hujusmodi, Ex duarum Æquationum Quadraticarum $z^2 + 2az - b = 0$, & $z^2 - 2az - c = 0$ in se invicem multiplicatione, Æquationem conficito Biquadraticam $z^4 = 4a^2 + b + c + 2z^2 + 2ac - 2ab \times z - bc$, cui terminus secundus deest, quamque huc Æquationi $z^4 = ez^2 + fz + g$ statuo æquipollere. Unde primo $4a^2 + b + c = e$ five $b = e - 4a^2 - c$. Secundò $2ac - 2ab = f$, hoc est, $2ac - 2ae + 2a^3 + 2ac = f$, five $c = \frac{f}{4a} + \frac{e}{2} - 2a^2$,

& inde $b = (e - 4a^2 - c) - \frac{f}{4a} + \frac{e}{2} - 2a^2$. Ter-

tio $-bc = g$, five $-\frac{f^2}{16a^2} + \frac{e^2}{4} - 2ea^2 + 4a^4 = -g$,

hoc est, $a^4 = \frac{1}{2} ea^2 - \frac{1}{4} ga^2 - \frac{1}{16} ea^2 + \frac{f^2}{64}$, quæ

Æquatio quasi Cubica est, ex Radice a^2 & notis vel as-
sumptis e, f, g constata. Ea verò Radix per Theorema
superius exhiberi potest, & eodem Calculo innotescent
ipsæ b & c . At Æquationum $z^3 + 2az - b = 0$ &
 $z^3 - 2az - c = 0$ Radices sunt $z = -a \pm \sqrt{a^2 + b}$

& $z = a \pm \sqrt{a^2 + c}$, five $z = -a \pm \sqrt{\frac{1}{2}e - a^2 - 4a^2}$,

& $z = a \pm \sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$, quæ proinde erunt Radices
Æquationis $z^4 = ez^2 + fz + g$; cognita videlicet a vel a^2
ex Æquatione $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea^2 + \frac{f^2}{64}$. Jam ut

Æquatio ista evadat universalis, & omnibus suis terminis
instructa, fac. $z = x - p$, eritque $x^4 - 4px^3 + 6p^2x^2$
 $- 4p^3x + p^4 = ex^2 - 2pex + p^2e + fx - fp + g$,

item & $x = p - a \pm \sqrt{\frac{1}{2}e - a^2 - \frac{f}{4a}}$, & $x = p + a \pm$

$\sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$. Tandem concinnitatis & compendii
gratia, fac. $e = 2q + 2p^2$ & $f = 8r$; tum $x^4 - 4px^3$

$+ 4p^2x^2 = 2qx^2 - 4pqx + 2p^2q + p^4 + 8rx - 8pr + g$,

$x = p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}}$, $x = p + a \pm$

$\sqrt{p^2 + q - a^2 + \frac{2r}{a}}$, & $a^6 = p^2 + q + a^4 - \frac{1}{4}g + \frac{1}{4}p^4$

$+ \frac{1}{2}p^2q - \frac{1}{4}q^2 + a^2 + r^2$. Denique fac $g = 4s - q^2$

$+ 8pr - p^4 - 2p^2q$, & fiunt Æquationes præcedentes

$x^4 = 4px^3 + 2qx^2 + 8rx + 4s$ & $a^6 = p^2a^4 - 2pra^2 + r^2$.

$-4p^2 - 4pq - q^2 + q - s$

Scilicet omnia evadunt ut supra sunt posita.

§ 3. Hactenus de *Æquationum Cubicarum & Biquadraticarum Resolutione Analytica*. Quoniam autem earundem *Effectio Geometrica* per Parabolam vulgò tradi solet, & nonnullis in pretio est, ipsam *conspiciat*, & quidem universalius, non pigebit hic exhibere.

Data *Æquatione* quavis vel Cubica vel Biquadratica, instituenda est comparatio inter terminos ejus, terminosque respondentes hujus *Æquationis*

$$x^4 = \frac{2p}{q} x^3 + \frac{4pr}{q} x^2 + \frac{2p^2}{q} x + p^2, \text{ quo pacto facile satis}$$

— 4r	— 4r ²	— $\frac{2p^2}{q}$	— q ²
	+ 2s	+ 4rs	— s ²
	— 1	— 2q	+ t ²

eruentur ipsæ p, q, r, s, t; earum interim unâ aliquâ ut-
cunque pro lubitu assumptâ. Tum in Parabola quavis
data AVB, cujus Vertex principalis V, Axis VS, & Axis

A geometric diagram of a sphere with several points and lines. The sphere is represented by a circle with a vertical line through its center. Points labeled include A, B, C, D, E, G, H, I, K, L, O, P, R, S, T, U, V, W, X, Y, Z. Lines connect these points, forming a complex geometric structure. The diagram is likely a technical drawing from a historical text, possibly related to astronomy or geometry.

perpendicularis VT ; capiatur $VS = p$ versus interiora Parabolæ, & in angulo SVT inscribatur $ST = q$, quæ producta Parabolam secet in punctis binis N & O . Bifecetur ON in M , & per M agatur MA Axi parallela & Parabolæ occurrens in A . Ipsi ON parallela ducatur AL , ut sit AL Latus rectum Parabolæ ad Diametrum AM , sitque hæc eadem Unitas. In AL (utrinque si opus est producta) capiatur $AG = r$, & à puncto G ducatur GR Axi parallela, quæ Parabolam secet in B , à quo capiatur $BR = s$. A novissime invento puncto R ducatur RE ipsi VT parallela & æqualis, quæ sinistram versus jaceat respectu ipsius R si q sit quantitas affirmativa, at versus dextram si q sit negativa. Atque idem de ipsis AG & BR intelligatur, quæ ad contrarias itidem partes duci debent, si modò valores ipsarum r & s prodeant negativi. Denique Centro E & Radio $EC = t$ describatur Circulus CK° , qui Parabolam in totidem secabit punctis, quot sunt Æquationis datæ Radices reales. Etenim à punctis istis C , K , &c. ducantur CP , κn , &c. ipsi ST parallela, & ad rectam GR (si opus est productam) terminatae, eritque harum quævis x , seu Æquationis datæ Radix quæsitæ; eæ scilicet ad dextram jacentes erunt Radices affirmativæ, quæ verò ad sinistram sunt posita erunt Radices negativæ. Punctum contactûs, siquod fuerit, hic sumitur pro intersectionis punctis duobus ad invicem vicinissimis.

Inter Æquationes Cubicas & Biquadraticas ita constructas hoc tantum intercedit discriminis, quòd in prioribus, ob terminum ultimum in præcedente Æquatione deficientem, semper sit $p^2 - q^2 - s^2 + t^2 = 0$, sive $t = \sqrt{s^2 + q^2 - p^2}$. Igitur Centro E & Radio $EB (= \sqrt{BRq + (ERq)STq - VSq})$ descripro Circulo CK° , Radicum una CP in priori constructione in nihilum abit.

Hæc autem demonstrantur ad modum sequentem. Mamentibus jam constructis, & producta CP si opus est, donec secat AM in H , erit CH Ordinata Parabolæ ad Diametrum

metrum AH, & proinde $CHq = AL \times AH = AH$, ob
 $AL = 1$. At $CH = CP + AG$, & $AH = GB + BP$, &
 propterea $CPq + 2AG \times CP + AGq = GB + BP$; sed
 ob naturam Parabolæ erit $AGq = GB$, unde $CPq + 2AG$
 $\times CP = BP$. Jam à puncto C ad ipsam BP demittatur
 norma s CD, quæ occurrat etiam ipsi EI, ad BP actæ pa-
 rallelæ, in puncto I. Propter similia Triangula CDP &
 TVS, erit $DP = \frac{VS \times CP}{ST}$ & $CD = \frac{VT \times CP}{ST}$, & pro-

$$\text{inde } CPq + 2AG \times CP = BP = DP + BD = \frac{VS \times CP}{ST}$$

$$+ BR - IE, \text{ five } CPq + 2AG \times CP - \frac{VS}{ST} CP - BR$$

$$= -IE. \text{ At } IEq = CEq - CIq = CEq - CDq \\ - VTq - 2CD \times VT = CEq - \frac{VTq \times CPq}{STq} - VTq$$

$$- \frac{2VTq \times CP}{ST} = (\text{ob } VTq = STq - SVq) CEq - CPq$$

$$+ \frac{SVq}{STq} CPq - STq + SVq - 2ST \times CP + \frac{2SVq}{ST} CP,$$

$$\text{quæ igitur æqualis erit Quadrato ex Latere } CPq + 2AG \\ \times CP - \frac{VS}{ST} CP - BR. \text{ Atque hæc } \text{Æquatio ad termi-}$$

nes p, q, r, s, t revocata ipsissima fit Æquatio proposita.

Hinc liquet, quòd eadem quævis Æquatio Biquadratica
 innumeras per Parabolam constructiones fortiri possit, pro
 indefinito valore quantitatis istius, quam ad arbitrium assu-
 mi posse jam diximus. Sed casus est simplicissimus faciendo
 $VS = p = 0$, & migrat constructio, si rem ipsam spectes,
 in vulgarem istam, in qua Radicum representatrices
 rectæ CP, &c. sunt ad Axem perpendiculares. Æquatio
 autem fit $x^4 = -4rx^3 - 4r^2x^2 + 4rsx - q^2$, quæ facile

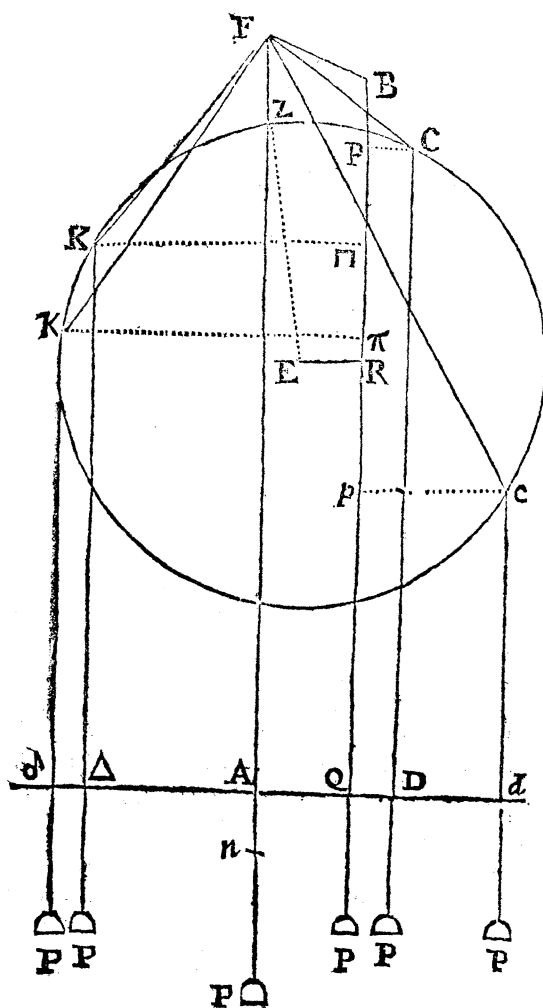
$$\begin{array}{rcccl} & + 2s & = & 2q & - s^2 \\ & - r & & & + t^2 \end{array}$$

construitur ut supra.

§ 4. Sed ne Parabolæ descriptio Organica difficilis nimium videatur, in promptu est Artificium quoddam Mechanicum, opes Fili penduli pondere instructi peractum, cujus auxilio quam exactissime & facillime Aequatio novissima construi potest, & proinde Aequationum quarumcunque Cubicarum & Biquadraticarum Radices inveniri; idque sine ullo linearum ductu nisi Rectarum & Circuli. Constructio autem, quam appellare libet *Mechanicam*, est ad hunc modum.

Contra Parietem erectum, vel planum aliud quodvis Horizonti perpendiculare, ad punctum aliquod F suspendatur filum tenuissimum & flexile FP ; pondere quovis P ad extremitatem P appenso. In hoc filo notetur punctum aliquod N , à puncto suspensionis F satis remotum; vel filo parvulus, si id mavis, innectatur Nodus N . Et sumpta utcunque NO pro Unitate, ad punctum medium A ducatur (in plano prædicto) recta AQ Horizonti parallela, & utrinque quantum satis producta. Hisce generaliter paratis, pro particulari jam applicatione fac $AQ = r$, ipsis q, r, s, t , ut sæpius inculcatum, vel Arithmetice vel Geometricè, pro datæ cujusvis Aequationis

ex-



exigentia, in *Æ*-
quatione novissi-
ma prius deter-
minatis. Tunc A-
cu vel Stylo tenu-
issimo, aut etiam
cuspidē Circini
admodum gracili,
flectatur filum à
loco suo ad pun-
ctum quoddam
B, ita ut punctum
N cadat in no-
vissime invento
puncto Q. In BQ
ab isto B capiatur
 $BR = s$, & in R
ad ipsam BR per-
pendicularis eri-
gatur $ER = q$.
Verum enimverò
istæ AQ, BR, RE
ad contrarias par-
tes ab earum ini-
tiis cadere debent,
si fortè valores
ipsarum r, s, q
prodeant negati-
vi. Denique in
puncto invento E

figatur Circini crus unum, & ad distantiam $EZ = t$ exten-
tum, agatur crus alterum in orbem, secumque circumducatur
filum FZP. Hac fili circulatione pondus P nunc ascendet
nunc descendet motu reciproco, ut & Nodus N nunc supra
rectam AQ extabit, nunc verò infra eandem deprimitur.
Quoties autem reperietur Nodus ille N in ipsa AQ, puta
in punctis D, d, Δ, Δ, ab scindet is rectas DQ, dQ, ΔQ, ΔQ.

quæ erunt *Æquationis* datæ Radices omnes reales ; hæ nempe ad dextram erunt Radices affirmativæ, illæ verò ad sinistram Radices negativæ. Demonstratio est manifesta ex præcedentibus, habita tantum ratione Parabolæ per puncta B, C, c, x, x transeantis. Nam posito F foco Parabolæ, (cujus distantia à Vertice est $\frac{1}{2}$ ON,) notum est quod lineæ omnes ut FB + BQ, FC + CD, &c, eandem ubique conficiant summam.

Atque ex principiis hic positis proclive erit Instrumentum haud inconcinnum & quantumvis accuratum fabricari, ejus beneficio hujusmodi *Æquationum* quarumcunque Radices nullo fere negotio inveniri possint, & præ oculis exhiberi. Hoc autem quilibet, si id Curæ sit, variis modis pro ingenio suo efficere potest, & de his jam satis.

III. *Æquationum* quarundam Potestatis tertiæ, quintæ, septimæ, nonæ, & superiorum, ad infinitum usque pergendo, in terminis finitis, ad instar Regularum pro Cubicis quæ vocantur Cardani, Resolutio Analytica.

Per Ab. De Moivre, R. S. S.

Sit n Numerus quicumque, y quantitas incognita, five *Æquationis* Radix quæsitæ, sitque a quantitas quævis omnino cognita, five ut vocant Homogeneum Comparisonis : Atque horum inter se relatio exprimat per *Æquationem*

$$ny + \frac{nn - 1}{2 \times 3} ny^3 + \frac{nn - 1}{2 \times 3} \times \frac{nn - 9}{4 \times 5} ny^5 + \frac{nn - 1}{2 \times 3} \times \frac{nn - 9}{4 \times 5} \times \frac{nn - 25}{6 \times 7} ny^7, \&c. = a$$

Ex